Single-photon transport properties in an optical waveguide coupled with a $\Lambda$-type three-level atom

XiaoFei Zang, Tao Zhou, Bin Cai, and YiMing Zhu

1Engineering Research Center of Optical Instrument and System, Ministry of Education and Shanghai Key Lab of Modern Optical System, University of Shanghai for Science and Technology, No. 516 JunGong Road, Shanghai 200093, China
2School of Mathematics and Physics, Shanghai University of Electric Power, No. 2103 PingLiang Road, Shanghai 200090, China
*Corresponding author: ymzhu@usst.edu.cn

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We investigate the single-photon transport properties in an optical waveguide coupled with a $\Lambda$-type three-level atom (ATLA) based on symmetric and asymmetric couplings between the photon and ATLA. The transmission and reflection coefficients of the single photon in such a hybrid system are deduced using the real-space approach. Symmetric and asymmetric bifrequency photon attenuators are realized by tuning the atom–photon couplings. The phase shift and group velocity delay of the transmitted single photon are also analyzed. © 2013 Optical Society of America

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1. INTRODUCTION

Single-photon and multiphoton transport properties in waveguide quantum electrodynamics (WQED) systems coupled with atoms or quantum dots (QDs) have attracted the full attention of many researchers [1–22]. In the past few years, many studies were focused on the coherent transport properties of single photons in various WQED systems, and many potential applications, such as single-photon switches, single-photon transistors, and generating entangled states between two QDs have been investigated [23–27]. Among these WQED systems, an optical waveguide embedded with a two-level atom is one of the most typical systems to control single-photon transport properties. For example, Shen and Fan have studied the single-photon transport properties in a one-dimensional waveguide coupled with a two-level atom and found that the single photon was completely reflected when the frequency of the incident photon was equal to the transition frequency of the two-level atom [26, 27]. In this case, the two-level atom can be considered as a completely reflecting mirror of the resonant photon due to the symmetric couplings between the photon and the two-level atom. However, the situation is very different from the above case for asymmetric atom–photon couplings. Recently, Yan et al. have proposed a method to realize the transmission of resonantly incident photons based on asymmetric couplings between the photon and a two-level atom [28]. They found that the transmission spectrum of the single photon could be well controlled by asymmetric atom–photon couplings and the frequency of the incident photon. As an application, a controllable single-frequency photon attenuator could be realized by controlling the asymmetric atom–photon couplings. Furthermore, multiphoton transport properties in a one-dimensional waveguide coupled with a two-level atom under the asymmetric atom–photon couplings were also investigated in [29]. The numerical results show that such a system could be served as an optical diode based on the strong and asymmetric atom–photon couplings.

Theoretical analyses of single-photon transport properties with asymmetric atom–photon couplings in the above systems are limited in a two-level atom model; however, an atom with more energy levels may lead to new physics on the coherent transport of the single photon. For example, Tsoi and Law investigated the single-photon scattering properties in a one-dimensional waveguide embedded with a chain of $N$-polarized $\Lambda$-type three-level atoms (ATLAs) [30]. They found that an incident photon with an unknown polarization could be converted into a specified one with higher transmission than that in polarizers obeying Malus’s law. In this paper, we study the single-photon transport properties in an optical waveguide coupled with an ATLA with symmetric and asymmetric atom–photon couplings. Our theoretical results demonstrate that new phenomena, such as symmetric and especially asymmetric bifrequency photon attenuators can be obtained, which are fundamentally different from the case of the two-level atom system [28]. Although the symmetric bifrequency photon attenuators have been studied by controlling the asymmetric couplings between the single photon and a $V$-type three-level atom [31], the asymmetric bifrequency photon attenuators cannot be observed in such a hybrid system. In particular, the phase shift and group velocity delay of the transmitted single photon in our hybrid system are also investigated.

2. THEORETICAL MODEL

In Fig. 1(a), we consider an optical waveguide coupled with an ATLA [Fig. 1(b)]. The total Hamiltonian of such a hybrid system can be described as
where \( H_w \) and \( H_{\text{atom}} \) describe the propagation of a single photon in the optical waveguide and ATLA, respectively. \( H_{\text{int}} \) is the atom–photon coupling term. In this hybrid system, the incident photon has a linear energy \( \omega \), momentum \( k \), and dispersion relation \( \omega = v_g k \), where \( v_g \) is the group velocity of the incident photon). Equation (1) can be written as

\[
H_w = -i v_g \int dx \left[ C_R^+(x) \frac{\partial}{\partial x} C_R(x) - C_L^+(x) \frac{\partial}{\partial x} C_L(x) \right],
\]

where \( C_R^+ \) and \( C_L^+ \) are the bosonic creation (annihilation) operators of the right-moving and left-moving photons at position \( x \), respectively. \( \sigma_{ij} \) (\( i, j = 0, 1, 2 \)) is the dipole transition operator between \( |i\rangle \) and \( |j\rangle \) [see Fig. 1(a)]. \( \gamma_1 \) and \( \gamma_2 \) describe the energy loss in [1] and [2] (here, we assume that the corresponding energy of the ground state is \( \hbar \omega_0 = 0 \)).

The atom–photon couplings can be written as

\[
g_{1,2} = \frac{2\pi \hbar}{\omega_{p1(p2)}} \frac{1}{\sqrt{\Omega_1 \cdot \Omega_2 \cdot E_{p1(p2)}}},
\]

where \( \Omega_1 \) and \( \Omega_2 \) are the corresponding energy eigenvalue of the ground state is \( \hbar \omega_0 = 0 \).

where \( M = \omega - \omega_1 + \Delta + i \gamma_2 \) and \( N = \omega - \omega_1 + i \gamma_1 \). Obviously, single-photon transport properties are decided by these parameters.

In what follows, we will discuss the coherent transport properties of the single photon by controlling these parameters, and many new phenomena are obtained.

\[
H_{\text{int}} = \int dx \left[ g_{1,2} \delta(x) [C_R^+(x)\sigma_{01} + C_L(x)\sigma_{10}] + g_2 \delta(x) [C_R^+(x)\sigma_{01} + C_R(x)\sigma_{10}] \right],
\]

The stationary eigenstate for the Hamiltonian [Eq. (1)] can be expressed as

\[
|E_k\rangle = \int dx [\phi_L(x)C_R^+(x) + \phi_L(x)C_L^+(x)]|0,0\rangle + a_1|0,1\rangle + a_2|0,2\rangle.
\]

The single photon comes from the left side, the wave function of \( \phi_{L,R} \) can be described as

\[
\delta = r \exp(-i k x)\theta(-x),
\]

where \( r \) and \( t \) are the reflection and transmission coefficients, respectively. We can obtain these two coefficients by solving eigenvalue equation of \( H_{\text{eff}}|E_k\rangle = E_k|E_k\rangle \). Here we assume \( \hbar = 1 \):

\[
t = \frac{MN(-\omega^2 + iM\Omega_1\Omega_2)^{1/4}}{M(-\omega^2 + iM\Omega_1\Omega_2)^{1/4}},
\]

\[
r = \frac{MN(-\omega^2 + iM\Omega_1\Omega_2)^{1/4}}{M(-\omega^2 + iM\Omega_1\Omega_2)^{1/4}}.
\]

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|E_k\rangle = \int dx [\phi_L(x)C_R^+(x) + \phi_L(x)C_L^+(x)]|0,0\rangle + a_1|0,1\rangle + a_2|0,2\rangle.
\]

The big QD can be considered as an ATLA, and it is coupled with nanowire waveguides. The radii of nanowires on both sides of the QD are asymmetric, so the asymmetric atom–photon couplings can be achieved experimentally (by changing the radii of the nanowires on both sides of the QD).

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\]
of the incident photon at $\omega = \omega_1 \pm \Omega/2$ is nonzero and increases with an increase in the ratio $g_1/g_2$. That is to say, these two side peaks at $\omega = \omega_1 \pm \Omega/2$ [Figs. 2(a1)–2(a4)] can be tuned from 1 to 0 by adjusting the right atom–photon coupling of $g_2$. In other words, a symmetric bifrequency all-optical attenuator can be realized by changing the asymmetric couplings between the photon and the ATLA, and it is very different from the single-frequency all-optical attenuator case in [28]. Therefore, we can control the transmission probability of the off-resonant photon from 1 to 0 by using the asymmetric atom–photon couplings. This kind of bifrequency all-optical attenuator is caused by the redistribution of energy and momentum of the incident photon after being scattered by the ATLA, due to the asymmetric atom–photon couplings [28].

The corresponding phase shifts imposed on the transmitted single photon are shown in Figs. 2(b1)–2(b4). In Fig. 2(b1), the transmitted photon is $\pi$-phase shifted when the frequency changes from $\omega = \omega_1 - \Omega/2$ to $\omega = \omega_1 + \Omega/2$, which may have a potential application in quantum phase gates [36,37]. For the asymmetric atom–photon couplings ($g_1 \neq g_2$), the phase shift of the transmitted photon is smaller than that of the symmetric case [see Figs. 2(b1)–2(b4)]. Based on the phase shift, one can also achieve the group delay of transmitted photon, which can be expressed as $\tau = \partial \phi/\partial \omega$ [21], as shown in Figs. 2(c1)–2(c4). When $\omega = \omega_1$, the incident single photon is completely transmitted with group delay of $\tau = 6$ fs (slow photon), as shown in Fig. 2(c1). The group delay is about 14 fs when the incident single photon is nearly (not) equal at $\omega = \omega_1 \pm \Omega/2$. [At $\omega = \omega_1 \pm \Omega/2$, a very large negative group delay (fast photon) is achieved, and it is not shown in Fig. 2(c1). This is because, at $\omega = \omega_1 \pm \Omega/2$, the transmission probability of the incident single photon is zero ($T = 0$)].

In the same way, taking advantage of the asymmetric atom–photon couplings, both the slow photon (positive group delay) and the fast photon (negative group delay) can also be realized by tuning the frequency of the incident single photon [see Figs. 2(c2)–2(c4)]. In Fig. 2(c2), we plot the group delay of the incident single photon with $g_2/2\nu_g = 0.012\nu_1$. For $\omega = \omega_1 \pm \Omega/2$, the incident single photon with $T = 0.38$ has a negative group delay of $\tau = -14$ fs, while the incident single photon with $T = 1$ at $\omega = \omega_1$ has a positive group delay of $\tau = 1.6$ fs. If we further increase the ratio of $g_1/g_2$ as shown in Figs. 2(c3) and 2(c4), both the slow photon and the fast photon can also be obtained. Comparing Figs. 2(c2)–2(c4), we can find that the group delay becomes smaller and smaller. This can be explained by noting that the variation of the phase shift decreases as $g_1/g_2$ increases. In other words, both the slow photon and the fast photon can be achieved in our ATLA system by controlling the asymmetric atom–photon couplings and the frequency of the incident single photon.

Now, we discuss the transmission spectrum, phase shift, and group delay of the single photon for $\Delta \neq 0$. The single-photon transmission spectrum with symmetric atom–photon couplings and $\Delta = 0.1\nu_1$ is shown in Fig. 3(a1). An asymmetric EIT-like transmission spectrum structure is achieved due to the detuning between level 1 and level 2 (the left dip is sharper than that of the right one). The incident single photon is completely reflected when $\omega = \omega_1 + (-\Delta \pm \sqrt{\Delta^2 + \Omega^2})/2$ and completely transmitted when $\omega = \omega_1 - \Delta$. For $\Delta \neq 0$, both the peak and the two side dips are shifted (due to the detuning $\Delta \neq 0$) when compared with the case of $\Delta = 0$. For the asymmetric atom–photon couplings, the transmission probability of the two side peaks at $\omega = \omega_1 + (-\Delta \pm \sqrt{\Delta^2 + \Omega^2})/2$ can also be tuned from 1 to 0. So, an asymmetric bifrequency...
all-optical attenuator can be obtained by controlling the atom–photon couplings [Fig. 3(a1)–3(a4)]. Meanwhile, the phase shift imposed on the transmitted single photon is also modulated by the atom–photon couplings, as shown in Figs. 3(b1)–3(b4). The variation of phase shift on the transmitted photon becomes smaller and smaller when increasing the

Fig. 3. (Color online) Transmission spectrum (a1)–(a4), phase shift (b1)–(b4), and group delay (c1)–(c4) for the transmitted single photon. The corresponding system parameters are $\omega_1 = 1\text{ ev}$, $\gamma_1 = \gamma_2 = 0$, $\Omega = 0.2\omega_1$, $\Delta = 0.1\omega_1$, $g_1^2/2\gamma_1 = 0.05\omega_1$, and $g_2^2/2\gamma_2 = 0.05\omega_1$ (a1)–(a4), $g_1^2/2\gamma_1 = 0.012\omega_1$ (a2)–(c2), $g_2^2/2\gamma_2 = 0.0035\omega_1$ (a3)–(c3), $g_2^2/2\gamma_2 = 0.00005\omega_1$ (a4)–(c4).

Fig. 4. (Color online) Transmission spectrum (a1)–(a4), phase shift (b1)–(b4), and group delay (c1)–(c4) for the transmitted single photon. The corresponding system parameters are $\omega_1 = 1\text{ ev}$, $\gamma_1 = \gamma_2 = 0.01\omega_1$, $\Omega = 0.2\omega_1$, $\Delta = 0.1\omega_1$, $g_1^2/2\gamma_1 = 0.05\omega_1$, and $g_2^2/2\gamma_2 = 0.05\omega_1$ (a1)–(c1), $g_1^2/2\gamma_2 = 0.012\omega_1$ (a2)–(c2), $g_2^2/2\gamma_2 = 0.0035\omega_1$ (a3)–(c3), $g_2^2/2\gamma_2 = 0.00005\omega_1$ (a4)–(c4).
ratio of $g_1/g_2$. The group delay of the transmitted photon is different from the case of $\Delta = 0$ [Figs. 3(c1)–3(c4)]. In Fig. 3(c1), the group delay nearly at $\omega = \omega_1 + (\Delta - \sqrt{\Delta^2 + \Omega^2})/2$ is much higher than that of the transmitted photon nearly at $\omega = \omega_1 + (\Delta + \sqrt{\Delta^2 + \Omega^2})/2$. This can be interpreted by noting that the variation of the phase shift on the transmitted photon nearly at $\omega = \omega_1 + (\Delta - \sqrt{\Delta^2 + \Omega^2})/2$ is much faster than that of the case nearly at $\omega = \omega_1 + (\Delta + \sqrt{\Delta^2 + \Omega^2})/2$. In addition, for the asymmetric atom–photon couplings, both the slow photon and the fast photon can also be realized, as shown in Figs. 3(c2)–3(c4).

In a real physical system, the unavoidable loss always brings photon leakage. Here, the atom loss ($\gamma_1 = \gamma_2 = 0.01\omega_1$) in such a hybrid system is considered. The transmission spectrum, phase shift, and group delay are shown in Figs. 4(a1)–4(c4). Figures 4(a1)–4(ad) show an imperfect all-optical and bifrequency attenuator due to the dissipative-dispersive effect (the two side peaks at $\omega = \omega_1 + (\Delta \pm \sqrt{\Delta^2 + \Omega^2})/2$ can just be modulated from 1 to nonzero). Moreover, the slow photon and the fast photon can also be obtained in the hybrid dissipative system by tuning the frequency of the incident single-photon and atom–photon couplings [Figs. 4(c1)–4(c4)].

4. CONCLUSION

In conclusion, we theoretically proposed a hybrid system containing an optical waveguide coupled with an ATLA to study the single-photon transport properties. By controlling the symmetric/asymmetric atom–photon couplings and the detuning ($\Delta$) between level 1 and level 2, symmetric and especially asymmetric bifrequency attenuators and slow/fast photons were realized. The impact of atom dissipation on the transport properties of the real physical system was discussed. This work may be helpful in manipulating light–matter interactions in WQED systems.

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