Design of cylindrical vector beams based on the rotating Glan polarizing prism

Qi Hu,1 Zhihua Tan,1 Xiaoyu Weng,1 Hanning Guo,1,* Yang Wang,2 and Songlin Zhuang1

1Engineering Research Center of Optical Instrument and System, Ministry of Education, Shanghai Key Lab of Modern Optical System, School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, 516 Jungong Rd, Shanghai 200093, China
2Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China
hmguo@usst.edu.cn

Abstract: Recently the cylindrically polarized beams have been gained highly attention in the fields of particle manipulation, material processing, nanoscale imaging, etc. So the methods to create the cylindrically polarized beams become more important. Here, based on the principle of the Glan polarizing prism, we design two types of the structures of the cylindrical polarization analyzer that can convert directly a linearly or circularly polarized beam into various cylindrical vector beams. The key optical element in the cylindrical polarization analyzer is the cylindrical polarizing prism with unique structure. We demonstrate the operating principle and the feasibility of the fabrication of the cylindrical polarization analyzer in detail. Analyses show that the cylindrical polarization analyzer designed by us not only have novel structures and excellent characteristics, such as the compact and stable structures, high extinction ratio, high polarization purity, no requirements on the mode and the wavelength of the incident light (only for the first type), not changing the intensity distribution of the incident light, and easily integrated into the optical systems, but also is easy to be fabricated, especially for the second type.

©2013 Optical Society of America

OCIS codes: (230.5440) Polarization-selective devices; (260.1180) Crystal optics; (260.5430) Polarization; (220.2740) Geometric optical design.

References and links

#184003 - $15.00 USD Received 22 Jan 2013; revised 7 Mar 2013; accepted 7 Mar 2013; published 15 Mar 2013 (C) 2013 OSA 25 March 2013 / Vol. 21, No. 6 / OPTICS EXPRESS 7343
In recent years considerable attention has been devoted to cylindrical vector beams including radially and azimuthally polarized beams due to their unique properties and potential applications [1–14]. For example, the focusing of radially polarized beam is used for particle manipulation [15, 16], Raman spectroscopy [17], second-harmonic generation [18], material processing [19], and nanoscale imaging [20–22] owing to the focusing properties with a larger axial field and a higher resolution than that for a linearly polarized illumination under the case of high numerical aperture system [1, 2]. Motivated by these applications, various methods for generating cylindrical vector beams have been proposed [23–36], having different degrees of polarization purity, complexity, efficiency, and cost. Generally, these methods can be classified into two types. One type involves direct generation from the laser cavity [23–25]. The other one generates the beam outside the laser cavity [26–36]. The former usually need one to specially redesign the laser systems allowing for customized tuning and

1. Introduction
modification of the output polarization or mode. However, it is not always viable or practical to insert the specially designed optical elements inside the resonator of existing lasers, especially in commercial systems, which are often viewed as “black boxes” by the end users. In contrast, the latter permits one to perform the polarization conversion outside the laser, which will bring great convenience and reduce cost in applications.

Among the methods generating the cylindrical vector beam outside the laser cavity, each has its advantages and disadvantages. For example, the method utilizing space-variant subwavelength diffraction gratings [26] requires tight fabrication tolerances for the visible spectrum and is sensitive to the illuminating wavelength. The method based on the combination of two orthogonally polarized beams [27–29] has stringent requirement on the modes of these two beams (only apply to the TEM₀₁ and TEM₁₀ modes) and the interferometric stability. The method of fiber [30] lacks intrinsic stability and are inefficient unless the input beam is already preshaped. The method based on the liquid crystal elements (e.g., liquid crystal spatial light modulators (SLM)) [31–33], theoretically, can generate a cylindrical vector beam with perfectly radial or azimuthal polarization distribution, but, because of the big size of pixel pitch and the fill factor in the liquid crystal elements, it is difficult to generate the cylindrically vector beam with high quality. Moreover, the SLM not only have the highest cost, but also is hardly integrated into the optical systems.

As we know, the linearly polarized beam is easily generated from the unpolarized beam by utilizing a sheet linear polarizer or a linear polarizing prism. Both the sheet linear polarizer and the linear polarizing prism are very simple optical devices with high conversion efficiency, no requirements on the mode and the wavelength of the incident light, and not changing the intensity distribution of the incident light. Especially for the linear polarizing prism, it has higher extinction ratio than the sheet linear polarizer and can be used for the incident light with high power. Obviously, if like the linear polarizing prism, there is a polarization analyzer that can convert directly the simple polarization, such as linear and circular polarization, into the cylindrical polarization only utilizing several geometrical optical elements (i.e., the prism, the reflector, and the lens), it will be very interesting and convenient in the practical applications. Hereafter, we call this polarization analyzer the cylindrical polarization analyzer.

In order to achieve this goal, Zhan and Leger [34] design a cylindrical analyzer based on the birefringent lens, where the birefringent material is cut such that its c axis is parallel to the optical axis and only one of the foci for the extraordinary and the ordinary beams is selected by the spatial filter. However, the conversion efficiency is very low because only the desired narrow annular section of the incident light is selected. Moreover, the intensity distribution of the incident light is changed, which is not permitted in many applications. For example, the intensity distribution the radially polarized higher-order Laguerre-Gaussian beam has significant effect on its focusing properties [14]. In 2007, Moh et. al [35] demonstrate the required Jones matrix of the cylindrical polarization analyzer when the incident light is circularly polarized. However, they do not provide the new corresponding cylindrical analyzer. In 2006, Shoham et. al [36] describe a system producing radially or azimuthally polarized beam and carry on preliminary experiment research, where the two conical reflectors and a cylindrical sheet of polarizing film are used. In our opinions, the concept presented by Shoham et. al [36] might be one of the most effective methods that like the linear polarizing prism, can construct the cylindrical polarization analyzer with excellent characteristics, such as the compact and stable structures, high conversion efficiency, high polarization purity, no requirements on the mode and the wavelength of the incident light, not changing the intensity distribution of the incident light, and easily integrated into the optical systems. Unfortunately, the concept presented by Shoham et. al [36] does not receive more attention and no further related works and products are reported. To our knowledge, the key reason is the fabrication of the cylindrical sheet of polarizing film with high extinction ratio and without a seam is almost impossible.

In this paper, based on the principle of the Glan polarizing prism, we design a cylindrically symmetrical polarizing prism with a coaxial hollow cylinder in order to take the...
place of the cylindrical sheet of polarizing film. For simplicity, this cylindrically symmetrical polarizing prism is hereafter called the cylindrical polarizing prism. We provide two types of the cylindrical polarization analyzer with novel structures. The structure of the cylindrical polarization analyzer in the first type is similar to that presented by Shoham et. al [36], but the novel and first reported cylindrical polarizing prism, especially that in the following second type, substitute successfully for the cylindrical sheet of polarizing film and overcome the fabrication difficulty. Based on the first type of the cylindrical polarization analyzer and the transmission matrix method, we derive the Jones matrix of the cylindrical polarization analyzer that is just the required Jones matrices of the azimuthal and the radial analyzers, predicated mathematically by Moh et. al [35]. In order to further reduce the fabrication difficulty of the cylindrical polarizing prism and make its structures of the first type more compact, the another cylindrical polarizing prism and cylindrical analyzer are designed successfully.

2. Design principle and discussions

![Diagram of the cylindrical polarization analyzer](image)

Fig. 1. Schematic (a) of the cylindrical polarization analyzer and its equivalent optical path (b). (c) Diagram of the structure of the cylindrical polarizing prism 5. Other optical elements are: the convex 45° conical reflectors 1 and 4, the concave 45° conical reflectors 2 and 3, the cylindrical polarizing prism 5, the spiral phase plate $P_1$, the $\lambda/2$ plates $P_2$ and $P_3$.

Our design principle is illustrated in Fig. 1(a), where all optical elements are coaxial and cylindrical symmetry and the optical elements inside the dashed rectangle consist the cylindrical polarization analyzer. Figure 1(a) only represents the cross section of the cylindrical polarization analyzer. A collimated vector beam passes through the convex conical reflector 1, the cylindrical polarizing prism 5, the concave conical reflectors 2 and 3 and the convex conical reflector 4 in turn. All reflecting planes of the convex conical reflectors 1 and 4 and the concave conical reflectors 2 and 3 make angles of 45° with the optical axis (same with the $c$ axis) of the cylindrical polarization analyzer. The key element of the cylindrical polarization analyzer is the cylindrical polarizing prism 5 separating the ordinary (o) and extraordinary (e) rays. Shown in Fig. 1(b), the cylindrical polarizing prism 5 is a birefringent crystal and is composed of the Part1 and the Part2. There are a convex conical transmitting surface and an inner cylindrical transmitting surface in the Part1. For the Part2, there are a concave conical transmitting surface and an outer cylindrical transmitting surface. As a whole, the cylindrical polarizing prism 5 has a coaxial hollow cylinder, which is like rotating...
a Glan polarizing prism with respect to its c axis. Like the Glan polarizing prism, the cylindrical polarizing prism 5 only permits either the o-ray or the e-ray to pass, so the generated cylindrical vector beam will have high polarization purity. The function of the concave conical reflector 2 is to make the divergent light reflected by the convex conical reflector 1 become parallel light again. The aim of the concave conical reflector 3 and the convex conical reflector 4 is to recover the intensity distribution of the incident light by adjusting the relative locations between them. In practical applications, the combination of the concave conical reflector 3 and the convex conical reflector 4 can also be used to construct annular beam.

In the following, we will derive the Jones matrix of the cylindrical polarization analyzer given by Fig. 1(a) to prove its feasibility. For the sake of analysis conveniently, the equivalent optical path of the cylindrical polarization analyzer is shown in Fig. 1(c) where the cylindrical polarizing prism 5 is not considered temporarily. For a treatment of the refraction and reflection that occurs at the interface, it is convenient to decompose the electric vector of the ray into s- and p-polarized vector components, $e_s$ and $e_p$, with respect to the meridional plane, respectively, and to rotate the coordinate system so that the new coordinate system will contain components in the $(p, s, z)$ system. This coordinate system is defined in such a way that $e_s = 0$. The meridional plane is defined as the plane consisting of the incident ray and the corresponding reflected ray. The $s$- and $p$-polarized vectors are perpendicular and parallel to the meridional plane, respectively. As we know, when a ray traverses a interface, the vibration direction of the $s$-polarized vector of the ray always keeps invariability for both the reflected and the refracted rays, whereas that of the $p$-polarized vector will rotate with the rotation of the propagation direction of the reflected and the refracted rays. Moreover, the $p$-polarized vector of the reflected ray will rotate angle of $180^\circ$ at the meridional plane. Namely, if the $(p, s, z)$ of the incident ray is right handed system, that of the reflected ray will be changed into the left handed system.

Figure 1(c) describes the variations of the $s$- and $p$-polarized vectors when a collimated vector beam passes through every optical elements of the cylindrical polarization analyzer. It is easy to derive that the Jones matrices $(T_1, T_2, T_3, T_4)$ of the convex conical reflectors 1 and 4 and the concave conical reflectors 2 and 3 are same and can be expressed as

$$
T_1 = T_2 = T_3 = T_4 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.
$$

The cylindrical polarizing prism 5 can be considered a linear polarizer for each incident ray. As the polarization directions of the o- and e-rays are separately consistent with the $s$- and $p$-polarized vectors of the incident ray, the Jones matrix of the cylindrical polarizing prism 5 is expressed as

$$
P_o = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},
$$

for the case of the e-ray reflected entirely and the o-ray transmitted, and

$$
P_e = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},
$$

for the case of the o-ray reflected entirely and the e-ray transmitted.

Shown in Fig. 1(c), the standard coordinate system of the cylindrical polarization analyzer is described by $x y z$ before the conical reflectors 1. For each ray, the coordinate system in its meridional plane before the convex conical reflectors 1 is defined as $x' y' z'$, where $\varphi$ is the
angle of the positive $x_\varphi$ axis with the positive $x$ axis and the $z_\varphi$ axis is consistent with the $z$ axis. So, the coordinate transformation for rotation around the $z$ axis from $xyz$ to $x_\varphi y_\varphi z_\varphi$ is

$$R = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix}. \quad (3)$$

Then, for the vector beam propagating along the $+z$ axis, the Jones matrices of the cylindrical polarization analyzer in the coordinate system $xyz$ are expressed as

$$M_o = R^T T_1 T_{z} P T_{z} T R, \quad (4a)$$
$$M_e = R^T T_1 T_{z} T P T R, \quad (4b)$$

where the superscript $T$ denotes the transpose. In terms of Eqs. (1)-(4), the Jones matrix of the cylindrical polarization analyzer can be obtained as following

$$M_o = \begin{bmatrix} \sin^2 \varphi & -\sin \varphi \cos \varphi \\ -\sin \varphi \cos \varphi & \cos^2 \varphi \end{bmatrix}, \quad (5a)$$
$$M_e = \begin{bmatrix} \cos^2 \varphi & \sin \varphi \cos \varphi \\ \sin \varphi \cos \varphi & \sin^2 \varphi \end{bmatrix}. \quad (5b)$$

The Jones matrices $M_o$ and $M_e$ of the cylindrical polarization analyzer are just the required Jones matrices of the azimuthal and the radial analyzers, predicated mathematically by Moh et al [35], respectively.

When a linearly or circularly polarized beam $E_i$ incients on the cylindrical polarization analyzer along the $+z$ axis, the corresponding output fields $E = ME_i$ are described as

For the $x$ linearly polarized beam $E_i = [1 \ 0]^T$,

$$E_o = -\sin \varphi [-\sin \varphi \ \cos \varphi]^T, \quad (6a)$$
$$E_e = \cos \varphi [\cos \varphi \ \sin \varphi]^T. \quad (6b)$$

For the $y$ linearly polarized beam $E_i = [0 \ 1]^T$,

$$E_o = \cos \varphi [-\sin \varphi \ \cos \varphi]^T, \quad (7a)$$
$$E_e = \sin \varphi [\cos \varphi \ \sin \varphi]^T. \quad (7b)$$

For the circularly polarized beam $E_i = [1 \ i]^T$,

$$E_o = i \exp(i \varphi) [-\sin \varphi \ \cos \varphi]^T, \quad (8a)$$
$$E_e = \exp(i \varphi) [\cos \varphi \ \sin \varphi]^T. \quad (8b)$$

It is noted that in generally, the radially and azimuthally polarized beams are called the cylindrical vector beams. The radial and the azimuthal polarizations are only dependent on the unit polarization vector $[\cos \varphi \ \sin \varphi]^T$ and $[-\sin \varphi \ \cos \varphi]^T$, respectively. The so-called cylindrical vector beam is defined as the beam with cylindrical symmetry in
polarization [6]. Therefore, the cylindrical vector beam is only determined by the cylindrical symmetry in polarization and is irrelevant to the amplitude and the phase modulation in its complex electric field. In terms of this point, although the sine or cosine amplitude modulation or the vortex phase \( \exp(i \phi) \) modulation exist naturally in Eqs. (6)-(8), the vector beam generated by the cylindrical polarization analyzer shown in Fig. 1(a) is still called the cylindrical vector beam. Equations (6)-(8) show that, for the case of the e-ray reflected entirely and the o-ray transmitted, the azimuthally polarized beam can be generated by the cylindrical polarization analyzer. For the case of the o-ray reflected entirely and the e-ray transmitted, the radially polarized beam can be generated by the cylindrical polarization analyzer.

In some cases, this natural modulation for amplitude or phase is beneficial. For example, Eq. (8) represents an azimuthally polarized beam with the vortex phase \( \exp(i \phi) \) that can generate sharper focus than other polarized beam including the usually radially polarized beam [8, 10]. When the incident beam is circularly polarized, to obtain a usually cylindrical vector beam [i.e., no vortex phase \( \exp(i \phi) \)], a spiral phase plate with the opposite helicity \( \exp(-i \phi) \) can be added at the exit [see the element \( P_1 \) in Fig. 1(a)] to compensate for the geometric phase [6, 35]. This type of spiral phase plate is available commercially [37].

In addition, it is inconvenient and will increase the cost for the end users and the vendors to fabricate two types of the cylindrical polarization analyzer (i.e., the azimuthal analyzer \( M_\theta \) and the radial analyzer \( M_r \)). A simple solution to this problem is to only fabricate the radial analyzer. The azimuthal polarization can be obtained through the polarization rotator using two cascaded \( \lambda/2 \) plates [see \( P_2 \) and \( P_3 \) in Fig. 1(a)]. If the angle between the fast axes of the two cascaded \( \lambda/2 \) plates is \( \Delta \phi \), the Jones matrix of this combination can be expressed as [6]

\[
T = \begin{bmatrix}
\cos(2\Delta \phi) & -\sin(2\Delta \phi) \\
\sin(2\Delta \phi) & \cos(2\Delta \phi)
\end{bmatrix},
\]

which is a pure polarization rotation function independent of the incident polarization. The polarization rotation angle can be tuned by adjusting the angle between the fast axes of the two plates.

So far we have derived the Jones matrix of the cylindrical polarization analyzer shown in Fig. 1(a) and demonstrated its feasibility theoretically. As the structure of the cylindrical polarization analyzer shown in Fig. 1(a) is similar to that presented by Shoham et. al [36], except for the cylindrical polarizing prism instead of the cylindrical sheet of polarizing film and the adding optical elements (the concave conical reflector 3 and the convex conical reflector 4) used to recover the intensity distribution of the incident light, and Shoham et. al carry on preliminary experiment research, we think the design of the cylindrical polarization analyzer shown in Fig. 1(a) is feasible in practice despite not provide any experimental results in this paper. As the fabrication of the cylindrical sheet of polarizing film with high extinction ratio and without a seam is almost impossible, the cylindrical polarization analyzer designed by Shoham et. al hasn't received more attention until now. The cylindrical polarizing prism designed by us is based on the principle of the Glan polarizing prism, so this cylindrical polarizing prism can offer ultra high extinction ratio like the Glan polarizing prism and its fabrication is more feasible than that of the cylindrical sheet of polarizing film.

Of course, we also note the fabrication of the cylindrical polarizing prism 5 shown in Fig. 1(b) is possible, but is not easy. Shown in Fig. 1(b), the cylindrical polarizing prism 5 is composed of the Part1 and the Part2. There are a convex conical transmitting surface and an inner cylindrical transmitting surface in the Part1. For the Part2, there are a concave conical transmitting surface and an outer cylindrical transmitting surface. The convex conical and the inner cylindrical transmitting surfaces of the Part1 must be parallel to the concave conical and the outer cylindrical transmitting surfaces of the Part2, respectively. Moreover, all surfaces
need to have optical quality with wavelength precision. These requirements will increase the fabrication difficulty of the cylindrical polarizing prism 5 shown in Fig. 1(b).

In order to further reduce the fabrication difficulty of the cylindrical polarizing prism shown in Fig. 1(b) and make the structures of the cylindrical polarization analyzer more compact, we design another cylindrical polarizing prism and cylindrical polarization analyzer (see Fig. 2). The basis for the design of the new cylindrical polarization analyzer is to reduce the numbers of the optical elements with curved surface, especially for the elements with concave conical surface, because to our knowledge, the convex conical optical element is available commercially, but the concave conical optical element is not. The fabrication of the concave conical optical element is more difficult than that of the convex conical optical element.

![Diagram of the cylindrical polarizing analyzer](image)

Fig. 2. (a) Schematic of the cylindrical polarization analyzer. (b) Diagram of propagating procedure of the o- and e-rays when the incident light transverses the optical elements 5 and 2. Other optical elements are: the $45^\circ$ planar reflecting prism 1, the concave $\alpha$ [calculated by Eq. (12)] conical reflector 2, the concave $45^\circ$ conical reflector 3, the convex $45^\circ$ conical reflector 4, the cylindrical polarizing prism 5, the transparent optical plate 6, the spiral phase plate $P_1$, the $\lambda/2$ plates $P_2$ and $P_3$. The blue-black dot and the purple short line denote the polarization directions of the o and e-rays, respectively. Note that, except for the $45^\circ$ planar reflecting prism 1, all other optical elements are coaxial and cylindrical symmetry about the c axis.

Compared with Fig. 1(a), the cylindrical polarizing prism 5 in Fig. 2(a) is simplified as the convex $45^\circ$ conical transmitting prism with negative birefringent crystal. The c axis is its symmetry axis. The convex conical reflector 1 in Fig. 1(a) is changed into the $45^\circ$ planar reflecting prism 1 in Fig. 2(a). The angle of the reflecting plane of the concave conical reflector 2 with respect to its optical axis (same with the c axis) is $45^\circ$ in Fig. 1(a), whereas this angle of the concave conical reflector 2 in Fig. 2(a) is $\alpha$ calculated by the Eq. (12). The additional optical element 6 in Fig. 2(a) is a transparent optical plate used for the support of the cylindrical polarizing prism 5, the $45^\circ$ planar reflecting prism 1, and the convex conical reflector 4 in Fig. 2(a). The other optical elements 3, 4, $P_1$, $P_2$, and $P_3$ are same with those in Fig. 1(a).

The cylindrical vector beam is generated in the following manner. Incident light upon entering the device, is reflected by the planar reflecting prism 1 and will propagate along the c axis of the cylindrical polarizing prism 5 to the interface I. The o and e components of the incident light propagate co-linearly and the refractive index $n''$ of the e-ray is equal to the primary refractive index $n_o$ of the o-ray. Under the proper conditions (will be given later), both the o- and e-rays are reflected entirely at the interface I. The direction of the reflected e- and o-rays will deviate at interface I due to the birefringence of prism 5 [see Fig. 2(a)].
In addition, as the cylindrical polarizing prism 5 is a negative birefringent crystal, i.e., 
\[ n_s \leq n^* \leq n_e \] (\( n_e \) is the primary refractive index of the e-ray), the reflection angle \( \theta_e \) of the 
e-ray at the interface I is bigger than the incident angle \( \theta_o \) of the incident light. Therefore, the 
reflected o-ray at the interface I is still reflected entirely by the interface II, whereas the 
reflected e-ray at the interface I will be transmitted at the interface II. So, the cylindrical 
polarizing prism 5 in Fig. 2(a) has the same function with the combination of the convex 
conical reflector 1 and the cylindrical polarizing prism 5 in Fig. 1(a).

Through selecting the proper angle \( \alpha \) of the concave conical reflector 2, we can always 
make the light reflected by the concave conical reflector 2 propagate along the optical axis 
(same with the \( c \) axis) of the cylindrical polarization analyzer. Obviously, the cylindrical 
polarization analyzer shown in Fig. 2(a) has the same function with that shown in Fig. 1(a). 
Compared with the cylindrical polarization analyzer shown in Fig. 1(a), the fabrication of the 
cylindrical polarizing prism 5 in Fig. 2(a) is easier and more feasible because it is just a 
convex 45° conical transmitting prism with negative birefringent crystal.

The natural question arises whether the above propagating procedure of the o- and e-rays 
can happen. Let’s take the calcite for example. The calcite is a usual material used for the 
polarizing prism and its primary refractive indices are \( n_o = 1.625 \sim 1.698 \) and 
\( n_e = 1.475 \sim 1.504 \) for wavelength \( \lambda = 350 \sim 2000 \text{nm} \). Note that all parameters about \( n_o \) 
and \( n_e \) of the calcite are seen in [38]. Shown in Fig. 2(b), as the cylindrical polarizing prism 5 
is the convex 45° conical transmitting prism, the incident angle of the incident light is 
\( \theta_o = 45° \) that is always bigger than the total reflection angle of the o-ray at the interfaces I 
and II (about 36° ~ 38°). So, the o-ray is reflected entirely at the interfaces I and II. For the 
reflected e-ray at the interface I, the following relations are met

\[
\sin \theta_e = \frac{n_s n_e}{\sqrt{n_e \sin^2 \theta + n_s^2 \cos^2 \theta}}, \\
\theta = \pi - \theta_o - \theta_e, \\
\sin \theta_e \left( \frac{\pi}{2} - \theta_e \right) = n_o \sin \theta_e.
\]

where \( \theta_e \) is the reflection angle of the e-ray, \( \theta \) is the angle of the propagating direction of 
the reflected e-ray at the interface I with respect to the \( c \) axis, and \( n^* \) is the corresponding 
refractive index of the e-ray. At the interface II, there is

\[
n^* \sin \left( \frac{\pi}{2} - \theta_e \right) = n_o \sin \theta_e,
\]

where \( n_o = 1 \) is the refractive index in free space and \( \theta_e \) is the refraction angle of the e-ray at 
the interface II.

In terms of Eqs. (10) and (11), under the case of \( \lambda = 350 \sim 2000 \text{nm} \), it is easy to calculate 
that \( n^* = 1.477 \), \( \theta_e = 51.1° \), \( \theta_o = 68° \) for \( n_o = 1.625 \) and \( n_e = 1.475 \). For \( n_o = 1.698 \) 
and \( n_e = 1.504 \), there are \( n^* = 1.51 \), \( \theta_e = 52.8° \), and \( \theta_o = 65.6° \). The refraction angles of the e-ray 
at the interface II \( \theta_o = 68° \) and \( \theta_e = 65.6° \) mean that the e-ray can be transmitted at the 
interface II. Therefore, the cylindrical polarization analyzer shown in Fig. 2(a) is feasible.

We note that \( \theta_e \) is different for the various wavelengths of the incident light, which means 
that the output beam of the cylindrical polarization analyzer shown in Fig. 2(a) might be a 
little divergent or convergent when the angle \( \alpha \) of the concave conical reflector 2 is given.
The solution to this problem is to limit the range of the wavelength of the incident light for the cylindrical polarization analyzer. For example, for the visible lights with $\lambda = 400 \sim 780 \text{ nm}$, $n_{e} = 1.649 \sim 1.683$ and $n_{o} = 1.482 \sim 1.497$. We can calculate $\theta_{i} = 65.9^\circ$ for $n_{o} = 1.683$ and $n_{e} = 1.498$ (i.e., $\lambda = 400 \text{ nm}$) and $\theta_{i} = 66.6^\circ$ for $n_{o} = 1.649$ and $n_{e} = 1.482$ (i.e., $\lambda = 780 \text{ nm}$). For the average wavelength $\lambda = 588 \text{ nm}$, $n_{o} = 1.659$, $n_{e} = 1.486$, and $\theta_{i} = 66.3^\circ$. If the angle $\alpha$ of the concave conical reflector 2 is designed in terms of the average wavelength $\lambda = 588 \text{ nm}$, the divergent or convergent degree of the output beam is minimum and about $\pm 0.3^\circ$. In terms of the geometrical relations of Fig. 2(a), $\alpha$ can be calculated by the following formulae

$$\beta + \alpha = \pi/2,$$

$$\alpha = \frac{\pi}{4} - \frac{1}{2}\theta_{i},$$

namely,

$$\alpha = \frac{3\pi}{8} - \frac{1}{2}\theta_{i},$$

where $\beta$ is the incident angle at the surface of the concave conical reflector 2. For $\lambda = 588 \text{ nm}$ ($\theta_{i} = 66.3^\circ$), $\alpha = 43.3^\circ$.

The above analyses show that the second type of the cylindrical polarization analyzer [i.e., Fig. 2(a)] is feasible and its fabrication is easier than that of the first type. However, compared with the second type, one major advantage of the first type is that there are not divergent or convergent for the output beams with various wavelengths because the transmitted $e$-ray always propagates along the direction normal to the surfaces of the cylindrical polarizing prism 5 [see Fig. 1(a)]. The other advantage of the first type is that it allows for a safe separation of the back reflected light with a suitable beam stop due to the Glan polarizing prism design. For the second type, as the rejected $o$-ray will return directly back to the laser and is azimuthal polarized, one had better use the polarization independent optical isolator before the laser to optically isolate the rejected $o$-ray.

3. Conclusion

In conclusion, based on the principle of the Glan polarizing prism, we design two types of the structures of the cylindrical polarization analyzer. It should be noted that, to our knowledge, the fabrication of the cylindrical sheet of polarizing film with high extinction ratio and without a seam is almost impossible, so the cylindrical polarization analyzer designed by Shoham et al hasn't received more attention and no further related work or product are reported. Therefore, the structure of the first type of the cylindrical polarization analyzer is similar to that presented by Shoham et al [36], except for the cylindrical polarizing prism instead of the cylindrical sheet of polarizing film and the adding optical elements used to recover the intensity distribution of the incident light, but the uniquely designed cylindrical polarizing prism overcomes the fabrication difficulty of the cylindrical sheet of polarizing film. Through deriving the Jones matrix of the first type of the cylindrical polarization analyzer, we demonstrate that it can produce the cylindrical vector beam. In order to further reduce the fabrication difficulty of the cylindrical polarizing prism and make the structures of the cylindrical polarization analyzer in the first type more compact, the second type of cylindrical polarizing prism and cylindrical analyzer are designed successfully. The basis for the design of the second type of cylindrical polarization analyzer is to reduce the numbers of the optical elements with curved surface, especially for the elements with concave conical surface. We demonstrate the operating principle and the feasibility of the fabrication of the
second type of cylindrical polarization analyzer in detail. Analyses show that the cylindrical polarization analyzer designed by us not only have novel structures and excellent characteristics, such as the compact and stabilize structures, high extinction ratio and polarization purity, no requirements on the mode and the wavelength of the incident light (only for the first type), not changing the intensity distribution of the incident light, and easily integrated into the optical systems, but also is easy to be fabricated, especially for the second type. The work in this paper is important for the generation of the cylindrical vector beam with high polarization purity and high optical quality.

Acknowledgments

This work was supported by the National Basic Research Program of China (2011CB707504), the Leading Academic Discipline Project of Shanghai Municipal Government (S30502), the National Natural Science Foundation of China (61178079 and 61137002), the Fok Ying-Tong Education Foundation, China (121010), the Foundation for the Author of National Excellent Doctoral Dissertation of PR China (201033), and the Science and Technology Commission of Shanghai Municipality (STCSM) (11JC1413300).